Math for Modelers

Gustav Levine and C. J. Burke

Mathematical Model Techniques for Learning Theories. New York: Academic, 1972. Pp. xii + 288. $12.50.

Reviewed by M. Frank Norman

Gustav Levine, the first author, is Associate Professor of Psychology at Arizona State University. He received his PhD from Columbia University and first was a research scientist at the Creedmoor Institute for Psychobiologic Studies. He spent a postdoctoral fellowship year studying mathematical models with C. J. Burke, coauthor of the book here reviewed. Burke, a University of Iowa PhD, is holder of a Career Research Award from the NIMH and currently is Professor of Psychology and Statistics at California State College, *Hayward. His writings have appeared frequently in professional journals.*

The reviewer, M. Frank Norman, is Associate Professor of Psychology at the University of Pennsylvania. His PhD was obtained from Stanford University. In 1970–71 he was a Guggenheim Fellow at Rockefeller University. Norman is author of Markov Processes and Learning Models.

According to the Preface, "The overriding intent of this text is to develop in the reader the ability to translate a set of verbal statements into mathematical equations." Intended readers are psychology students and professional psychologists approaching the mathematical model literature for the first time. The only prerequisite is elementary algebra.

The Preface would lead us to believe that Levine's and Burke's main concern is the construction of mathematical learning models, and, indeed, a conscientious reader is likely to develop skills that will assist him in formulating models. However, the primary emphasis is not on 'making' models, but rather on 'solving' them. Most of the techniques referred to in the title are best viewed as methods of deriving predictions from previously formulated models. These methods are copiously illustrated by application to standard learning models. Thus the reader can become acquainted with these models at the same time he is learning how to work out formulas for their predictions. There are exercises at ends of chapters, and detailed solutions in the back of the book. These exercises give the student useful practice in applying the techniques described.

The plan of the book is considerably more attractive than its execution. In particular, the reader is likely to be discouraged by repetitive and occasionally pointless calculation. This is especially worrisome since the psychological background of the models considered is not developed sufficiently to permit the uninitiated reader to fully appreciate their importance. Instead of a general overview of the role of models in learning theory, we find, in the introductory chapter, a fussy discussion of variations in the usage of the terms "model" and "theory." No examples of models or theories are spelled out to clarify distinctions for the beginner.

The mathematical methods developed in later chapters are probability, summation of geometric series, solution of difference equations, matrix algebra, and Markov chains. All of the learning models considered in the book are probabilistic, and the reader must follow some rather intricate calculations, so a fairly thorough exposition of the elements of probability theory is required. The treatment given is seriously deficient in both scope and clarity. The following examples illustrate its defects. The term "event" is used for both elements and subsets of a sample space. This is likely to cause confusion because the text shifts back and forth between these two usages without adequate warning. Probabilities are "operationally understood" to be long-run sample frequencies on p. 14, but on pp. 16 and 17 certain probabilities are defined via simple frequency ratios. It is not clear whether these are intended to be short-run sample frequencies (in which case the definition is inaccurate) or population frequencies (in which case an explanation is in order). The distinction between sample means and expectations is similarly blurred. The variance of a random variable is not defined or explained. Instead, we are given a computational formula equivalent to $\sigma^2 = E(X^2) - E(X)^2$.

In contrast to the skimpy treatment of probability, the discussions of geometric series and difference equations are grossly inflated. For example, Table 2.1 lists the sums of no less than twelve trivial variations of the finite series $\Sigma_{k=0}{}^{n}C^{k}$, obtained by replacing the lower limit by 1 and 2, the exponent by $k - 1$, and C by $1 - c$. Table 2.2 gives the comparable sums for $n = \infty$. Five of these sums are derived from scratch in Figs. 2.7–2.11, and, when $\Sigma_{k=0}{}^{n-2}A^{k}$ comes up later (p. 62), it is calculated directly rather than referred to Table 2.1. The long chapter on difference equations details three different methods of solving the same simple linear equations. This makes the subject appear much more complicated than it really is.

The material on matrix algebra is quite adequate and is not duplicated in other books on learning models. The same is true of the detailed treatment of finite Markov chains, which brings together probability theory and matrix algebra.

The models used for illustrative purposes are the classical linear and stimulus sampling models, LaBerge's stimulus sampling model with neutral elements, Bower's all-or-none model, and certain two-stage models of Theios and his collaborators for avoidance conditioning. Bower's model is an ideal example since it is mathematically simple yet psychologically interesting, and it appears again and again in this book. The discussion of Theios's model, in connection with Markov chain theory, is also handled nicely.

On the other hand, the treatments of the older linear and stimulus sampling models are rather unsatisfactory. Since Levine and Burke assume that there is no variability in the sampling process of the stimulus sampling model, it is mathematically equivalent to the linear model. However, the authors treat the two models in different chapters using different notations, and describe the similarity between them incorrectly (pp. 124–125). The variance formulas on p. 117 are also erroneous, since they wrongly assume that subjects' responses on different trials are independent in the linear model. Finally, the linear model chapter contains several pages of calculations of second and higher moments of response probabilities, but it contains not one word to indicate the use of these quantities.

The final chapter gives a clear exposition of Greeno's and Steiner's important recent work on observable states and identifiable parameters for models with a small number of states of learning. This chapter, and the three that precede it (on matrix algebra, Markov chains, and avoidance conditioning) are much better than the three earlier chapters (on probability, difference equations, and linear models), to which most of the above criticisms apply.

There are other ways of deriving predictions from learning models than the exact algebraic methods exposited by Levine and Burke. Computer based methods, such as Monte Carlo simulation, and approximation techniques, like the expected operator approximations of Bush and Mosteller, are not even mentioned. This is most regrettable because only the very simplest models permit derivation of exact formulas for predictions, while simulation and approximation have much broader applicability.

In conclusion, the later chapters may be useful as supplementary reading in a mathematical models course, but Atkinson, Bower, and Crothers would be far better as a basic text. It covers a broader range of models and gives more psychological background and empirical evaluation. Furthermore, even though it is not as technique-oriented as the book under review, it is actually mathematically sounder and teaches more effectively many of the methods considered by Levine and Burke.